The Antenna Placement Problem for Mobile Radio Networks: An Evolutionary Approach

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Abstract

The antenna placement problem (APP) is an important task in the design of radio networks. We introduce a model that addresses cover, traffic demand, interference, different parameterized antenna types, and the geometrical structure of cells. The resulting optimization problem is constrained and multi-objective. We have developed an evolutionary algorithm, capable of dealing with more than 700 candidate sites in the working area. The results show that the APP is tractable.

Nowadays the APP is solved by experts, only supported by software tools for visualization and manipulation of network designs. The automatically generated designs enable experts to focus their efforts on the difficult parts of a design problem.

1 Introduction

Engineering of current mobile radio networks (mainly GSM at present) consists of several different tasks: traffic estimation, radio antenna positioning, broadcast control, frequency assignment etc. Within a selected geographic area where the radio network must be installed or extended, operators define the number of radio transmitters to be installed and the number of frequencies to be assigned to the area. These parameters are then used for the placement of antennas and the frequency assignment.

The purpose of the *Antenna Placement Problem (APP)* is to optimize the radio coverage of an area. The main objective of the *Frequency Assignment Problem (FAP)* is to minimize electromagnetic interference due to multiple use of frequencies in different parts of the network. Whereas FAP has been intensively investigated [Ple94, RC97, CM98a, CM98b], little has been done for APP [LHKC98, HV98]. The reason is that APP is much more difficult to model, and even a simple model – treating only coverage of the working area – can be shown

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to be NP-complete [ESW98]. Furthermore APP is a multi-objective problem with all its known difficulties.
Within the EU project *ARNO (Algorithms for Radio Network Optimization, IT Project 23243)* the APP has been investigated. In the paper we first describe the mathematical model used within ARNO (see also [RC98]). It consists of points defined on a grid, e.g. service test points, traffic test points and candidate sites. Radio transmission is modelled by a propagation loss matrix. The objectives are to minimize cost, to minimize the interference level and to have geometrically "nice" cells. These objectives are components of a cost function which guides the evolutionary search.
The outline of the paper is as follows. First we develop a mathematical description of the problem. In Section 3 the objectives and constraints of the problem are defined. Our evolutionary algorithm is presented in Section 4. Results from real world benchmarks are discussed in Section 5.

2 Antenna Placement Problem

Developing an appropriate model is one of the most difficult tasks in solving complex real world problems. The model for the APP used in this paper is the result of several loops of design, evaluation and redesign. The quality of solutions is assessed by two constraints, which deal with cover and traffic demand, and three objectives, which address economical and technical aspects. In order to define our APP-model we introduce the following concepts:

1. Input Data

- A set $\mathcal{R}$ of *Reception Test Points (RTP)*, given by coordinates (x,y).
- A set $\mathcal{S}$ of *Service Test Points (STP)*. A *signal quality threshold* S_q is assigned to each STP, usually -90 dBm.
- A set $\mathcal{T}$ of *Traffic Test Points (TTP)*. Each TTP carries *traffic demand*, measured in Erlang.
- A set $\mathcal{L}$ of coordinates of *Candidate Sites*.

Note that $\mathcal{T} \subset \mathcal{S} \subset \mathcal{R}$. Usually the RTPs form a rectangular grid. The position of a candidate site does not have to coincide with a RTP. Figure 1 shows an example of a data map. Candidate sites are displayed as black circles, RTPs are colored white, STPs light grey and TTPs are colored in darker shades of grey, according to increasing traffic demand.

- A *Propagation Loss Matrix* PLM_i for each candidate site L_i defines the signal losses (in dBm) from site L_i to all RTPs.
- An *Angle of Incidence Matrix* AIM_i for each candidate site L_i defines the vertical angles at which the RTPs appear to site L_i.

2. Antenna Types

- omnidirectional antenna (OD):

  Parameters: Power, ranging from 26 dBm to 55 dBm in steps of 1 dBm.

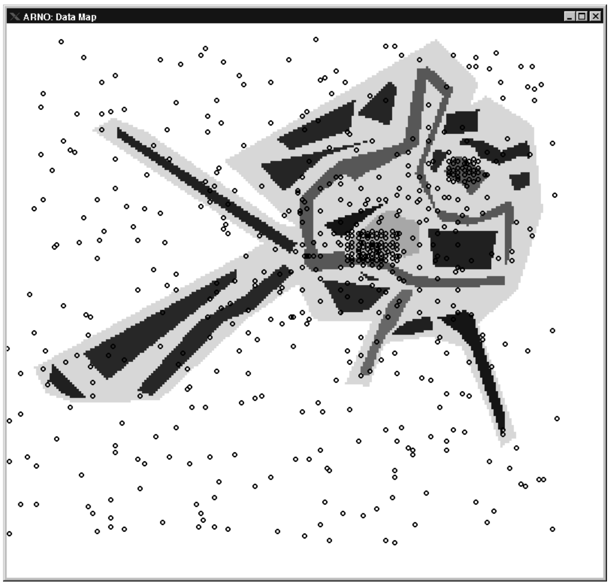

Figure 1: Data Map

- large directive antenna (LD):

  Parameters: Power, ranging from 26 dBm to 55 dBm. Azimuth, varying in steps of 1°. Tilt, ranging form 0° to -15° in steps of 1°.

- small directive antenna (SD):

  Parameters: same as for large directive antenna.

All antenna types can handle a traffic of up to 43 Erlang. Associated with an antenna type are *antenna diagrams*: a vertical diagram (VDIAG) for omnidirectional antennas and vertical and horizontal diagram (HDIAG) for the directive ones. These diagrams define the relation between the *radiant signal loss* (in dBm) and the radial deviation from the signal main axis, measured in degree. Furthermore each antenna type has a specific *antenna gain (G)* and a specific *antenna loss (A)*, measured in dBm:

Antenna Type	G	A
OD	11.15	7.00
LD	15.65	7.00
SD	17.15	7.00

Table 1: Antenna Gain and Antenna Loss

AT_{ij} denotes the jth antenna at site L_i. Accordingly, $Power_{ij}$, G_{ij}, ...denote the respective parameters of AT_{ij}, and (u_i, v_i) are the coordinates of site L_i. The field strength $F_{ij}(x, y)$ of antenna AT_{ij} at reception test point $r = (x, y)$ is computed according to the following formula:

$$F_{ij}(x,y) = Power_{ij} + G_{ij} - A_{ij} - PLM_i[x,y] - VDIAG[AIM_i[x,y] - Tilt_{ij}]$$
$$-HDIAG[\tfrac{180^\circ}{\pi} \cdot atan2(y - v_i, x - u_i) - Azimuth_{ij}] \ .$$

VDIAG and HDIAG are the diagrams associated with the antenna type of AT_{ij}. If AT_{ij} is omnidirectional, we neglect the HDIAG term and set $Tilt_{ij}$ to zero. $atan2(x,y)$ is similar to $arctan(x/y)$, except that the signs of both arguments are used to determine the quadrant of the result.
A *configured antenna* is a pair defining an antenna type and a complete list of parameter values for that antenna type, e.g. (LD, [Power = 50 dBm, Azimuth = 90°, Tilt = -5°]). A *configured site* is a site carrying at least one configured antenna. A configured site can carry either one omnidirectional antenna or at most three directive antennas. A *solution* for an APP is a list of configured sites. The *cell* C_{ij} of an antenna AT_{ij} is the set of STPs s receiving the best signal (i.e. strongest field strength) from AT_{ij}, provided that the signal is above the signal quality threshold of s. The *traffic load* of an antenna AT_{ij} is the sum of the traffic demand of all TTPs contained in the cell C_{ij} of B_{ij}. An antenna with traffic load above 43 Erlang is called *overloaded*. The percentage of handled traffic demand of a solution is called *traffic hold*:

$$\mathit{TrafficHold} = \frac{1}{\mathit{TotalTraffic}} \sum_{AT \in \mathcal{AT}} \min(\mathit{TrafficLoad}(AT), 43 \text{ Erlang}),$$

where *TotalTraffic* is the summed traffic demand of all TTPs and $\mathcal{AT}$ is the set of all antennas. The percentage of lost traffic demand is called *traffic loss*, i.e. *TrafficLoss* = 1 - *TrafficHold*.

3 Constraints and Objectives

A *feasible* solution has to fulfill the following two constraints:

- **Traffic Constraint (TC):** The traffic load of an antenna is $\leq$ 43 Erlang,
- **Cover Constraint (CC):** Every STP receives at least one signal above its signal quality threshold.

Note that the constraints imply that all TTPs of a feasible solution are contained in a non-overloaded cell. Thus the traffic hold of a feasible solution is 100%. The next step is the definition of one or more objectives. From the point of design and evaluation of solutions it is highly desirable to have only one objective. But it is often very difficult to combine the different "quality-dimensions" of a solution into one single objective. So, we first introduce three "quality-dimensions" as objectives to evaluate solutions. For the evolutionary algorithm these objectives will be combined into one cost function, see section 4. The objectives are:

- **Site Cost:**

  $$SC = \text{number of used Sites}$$

- **Interference Level:**

  $$IL = \frac{1}{\mid \mathcal{S} \mid} \sum_{s \in \mathcal{S}} \sum_{F \in \mathcal{F}_s \setminus \mathcal{H}_s} \max(F - S_m, 0)$$

where $\mathcal{F}_s$ is the set of field strengths of all antennas at STP s. $\mathcal{H}_s$ is the *handover set*, consisting of the 4 strongest signals at STP s. S_m is a sensibility threshold, usually -99 dBm.

- **Cell Shape Factor:**

$$SF = \frac{1}{\mid Cells \mid} \sum_{C \in Cells} \frac{boundary(C)}{\sqrt{area(C)}}$$

where $boundary(C)$ is the number of boundary points of cell C, i.e. the number of $s \in C$ having a $s' \in \mathcal{S} \setminus C$ in their 8-neighborhood, and $area(c)$ is the number of interior points of C, i.e. the number of non-boundary points of C. This measure is inspired by a ratio widely used in physics: the ratio between the square root of the surface and the third root of the volume of a body. This ratio is called *shape factor*. It is dimensionless, scale invariant, and reaches its minimum value for a ball.

Site cost addresses the economical, *interference level* and *cell shape factor* address technical aspects of the APP. The intention of the interference objective is to make the frequency assignment problem (FAP) as simple as possible. The shape factor objective prefers geometrically well-formed cells, which is highly desirable for several reasons, e.g. minimization of drop-out probability. Within the ARNO project, the objective cell shape factor has not been used. Instead there was a "connectivity constraint" which ensures that all cells are topologically connected. But it turned out that this constraint has several disadvantages:

1. it is very difficult to fulfill,
2. it is very sensitive to small changes in design parameters,
3. connected cells can still be very irregular.

For these reasons we have replaced the connectivity constraint by the shape factor objective, which seems to avoid some or all of the above problems.

4 Evolutionary Approach

The above problem is solved by an evolutionary algorithm consisting of three phases:

- Initialization Phase
- Repair Phase
- Optimization Phase

This scheme provides a flexible base for the adaptation of the abstract meta-heuristic *Evolutionary Strategy* (ES) to the APP. A general introduction into the field of evolutionary search can be found in [Sch95] and [AL97]. The internal structure of the three phases is as follows:

1: Initialization Phase

INITIALIZE Network (guided by heuristic rules)

2: Repair Phase (repairs violated constraints)

REPEAT
  SELECT Repair Operator
  APPLY selected Repair Operator on Network
UNTIL Network is feasible **OR** Stop Condition

3: Optimization Phase (optimizes feasible network)

REPEAT
  SELECT Climb Operator
  APPLY selected Climb Operator on Network
  APPLY Local Repair on Network
UNTIL Stop Condition

We use two types of cost functions:

1. *Hard Cost Function*, measuring violation of constraints in the repair phase,
2. *Soft Cost Function*, measuring the cost of a feasible solution in the optimization phase.

Our cost functions are linear combinations of the single cost or penalty terms. For example:

$$HardCost = \phi_1 \cdot \mathit{TrafficLoss} \ + \ \phi_2 \cdot \mathit{UncoveredSTP} \ ,$$

where ϕ_1 and ϕ_2 are weight factors representing the relative importance of the single terms. *UncoveredSTP* denotes the number of uncovered STP. An example for a soft cost function is:

$$SoftCost = \psi_1 \cdot SC \ + \ \psi_2 \cdot IL \ + \ \psi_3 \cdot SF$$

4.1 Initial Solutions

In order to get a reasonable *initial solution* we exploit the *local structure* around a site. For this purpose we introduce the following notions:

traffic demand density (in area A):

sum of traffic demand in A divided by the area of A

candidate site density (in area A):

number of candidate sites in A divided by the area of A

Depending on these densities in the neighborhood of a site an *initial placement probability* for this site is computed. Then a configuration of antennas is placed at this site according to the computed probability. The user has different choices

for the configuration. Repeating this procedure for all sites generates an initial solution.
The formula for the placement probability reflects the local structure around a site by balancing the local traffic demand density and the local candidate site density. The formula is split into two parts because we have to deal with the cover constraint as well. Usually the traffic demand restricts the size of a cell. But if the demand density drops below a critical value (ρ^*_{TD}), the cell can become so large (with regard only to the traffic constraint) that the field strength does not reach the service threshold at the cell periphery. Hence in this case the placement probability should depend on the *maximal cell size* of a site and not on the local traffic demand.
This approach leads to 'good' *initial* solutions if the problem structure is not too irregular (e.g. strongly varying densities). For a given candidate site L_i let

ρ_{TD} = local traffic demand density $\qquad [\rho_{TD}] = \text{Erlang} \cdot \text{m}^{-2}$,

ρ_{L_i} = local candidate site density $\qquad [\rho_{L_i}] = \text{m}^{-2}$,

C_{max} = maximal traffic capacity $\qquad [C_{max}] = \text{Erlang}$,

A_{max} = maximal cell area $\qquad [A_{max}] = \text{m}^2$.

C_{max}, the maximal traffic capacity of site L_i, depends on the *placement policy.* The placement policy defines what configuration will be placed initially at a selected site. If an omnidirectional antenna is placed, then C_{max} is 43 Erlang. If the policy defines to place two or three directive antennas, C_{max} takes the values 86 and 129 Erlang, respectively. A_{max}, the maximal cell size of a site L_i, is defined as the area where the field strength of an omnidirectional antenna with maximal power — placed at site L_i — is above the service threshold. These are the parameters we need in order to compute the placement probability. A_{max} is computed by the following steps:

1. Place an omnidirectional antenna at site L_i with maximal power.

2. Let N be the number of STPs receiving a good signal from the placed antenna.

3. $A_{max} = N \cdot \Delta^2_{mesh}$ (Δ_{mesh} = width of square mesh)

Our formula for the *placement probability* is given by:

$$p = \begin{cases} \min\left(\frac{1}{C_{max}} \cdot \frac{\rho_{TD}}{\rho_{L_i}},\ 1\right) & ,\ \rho_{TD} \geq \rho^*_{TD} \quad \text{(critical constraint: traffic)} \\ \min\left(\frac{1}{A_{max}} \cdot \frac{1}{\rho_{L_i}},\ 1\right) & ,\ \text{otherwise} \quad \text{(critical constraint: cover)} \end{cases}$$

where $\rho^*_{TD} = C_{max}/A_{max}$.

For the estimation of the parameters ρ_{TD} and ρ_{L_i} we assume that the distributions of candidate sites and traffic demand are not too irregular. Thus we can use the ideal situation of a network of hexagon cells (see Figure 2) as orientation to derive our density estimators. There are a lot of other reasonable estimators, especially if the 'not too irregular' assumption fails. However, note that the initial placement probabilities need to be only approximately 'correct', because we are interested in an *initial* solution and not in a final one.

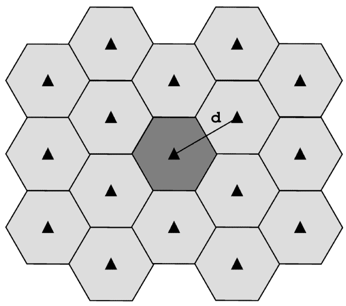

Figure 2: Hexagon Network

With regard to Figure 2 we use the following estimation procedures for the densities ρ_{TD} and ρ_{L_i} in the neighborhood of a candidate site L_i:

Local Candidate Site Density

1. Compute the distances from site L_i to its 6 nearest candidate sites: $d_1, .., d_6$

2. $d = \frac{1}{6} \sum_{i=1}^{6} d_i$

3. $A_{hexagon} = \frac{\sqrt{3}}{2} \cdot d^2$ (area of hexagon)

4. $\rho_{L_i} = \frac{1}{A_{hexagon}}$

The following estimator of traffic demand density ρ_{TD} uses the notion of the *bounding box* of a set $A \subset \mathbb{R}^2$. It is a rectangle

- with sides parallel to the coordinate axes,
- containing A,
- with minimal area.

We use the bounding box, because in our context not only the accuracy, but also the computational efficiency of an estimator is important.

Local Traffic Demand Density

1. box = bounding box of the 6 nearest candidate sites

2. $e_{box} = \sum_{t \in \mathcal{T} \cap box} e(t) \qquad (e(t) = \text{traffic demand at TTP } t)$

3. A_{box} = area of box

4. $\rho_{TD} = \frac{e_{box}}{A_{box}}$

4.2 Repair Phase

After finishing the initialization phase, the candidate solution is analyzed: if it violates a constraint, the design process enters the *repair phase*. The goal of the repair phase is to transform the candidate solution into a feasible one. The transformation is carried out by a number of *heuristic operators* or, for short, *heuristics*. The implemented heuristics are:

- **RepairTraffic**: tries to introduce new antennas in order to absorb traffic overload of nearby cells,
- **RepairHole**: tries to introduce new antennas in the neighborhood of cover holes,
- **DecreasePower**: decreases power of an antenna with traffic overload step by step until the overload vanishes or the minimal power is reached,
- **IncreasePower**: increases power of an antenna with traffic load $\ll$ 43 Erlang step by step until the maximal traffic load or the maximal power (55 dBm) is reached,
- **ChangeAzimuth**: changes the azimuth of an antenna by a random value,
- **ChangeTilt**: changes the tilt of an antenna by a random value,
- **DissipateTraffic**: tries to reduce traffic over- and underload of all antennas using a dissipation algorithm (see below).

The dissipation algorithm used in the DissipateTraffic-rule executes the following procedure:

1. Run through the list of antennas.
2. If an antenna with power > 26 dBm has traffic overload
   then decrease its power by 1 dBm.
3. Repeat until a run through the antenna list results in no change.

In many cases the effect of this rule is that traffic load peaks will dissipate over the whole network until there is no more traffic overload. This rule has proven to be very successful in eliminating traffic overload.
A *selection operator* chooses a heuristic operator from the above list, which is then applied to the network. Currently we use *uniform selection*, i.e. each heuristic is chosen with the same probability. The transformed network is only accepted if its hard cost is lower than or equal to the hard cost of the original one. This process will be repeated until a feasible solution results – then the design process enters the *optimization phase* – or the *termination criterion* (see 4.4) is reached. In the latter case the design process will be aborted.

4.3 Optimization Phase

The optimization phase has a similar structure to the repair phase. The two main differences are:

- A soft cost function instead of a hard cost function guides the search.
- If a network operator destroys feasibility of a solution, then it is tried to repair this new candidate solution in order to maintain feasibility. If feasibility cannot be restored, the new candidate solution will be discarded.

For the transformation of networks there are additional heuristics available:

- **RemoveWeakAntenna**: looks for an antenna with low traffic load or small cell size and deletes it,
- **RemoveAntenna**: deletes a random antenna,
- **RemoveWeakSite**: deletes all antennas of a weak site, e.g. a site with low traffic load,
- **RemoveSite**: deletes all antennas at a random site,
- **IncreaseCompactness**: looks for a cell with high shape factor and reduces power or increase tilt of the respective antenna,
- **ReduceIrregularities**: introduce new antennas guided by the irregularity measure (see below) in order to reduce regions with irregular geometrical and topological cell structures.
- **MinimizePower**: reduces power of all antennas by the same amount as long as no new uncovered STPs emerge. Thus the cell structure stays unaltered while the interference level is reduced.

One major problem in the optimization phase is the construction of networks with a reasonable cell topology and geometry. Due to irregular path loss matrices (reflecting irregular geographical structures) many cells tend to have an irregular shape or even get disconnected. Therefore we introduce a *local* measure for 'geometrical irregularity'. This measure guides the optimization process in order to reduce such irregularities. The local irregularity $I_r(\mathbf{x})$ – defined in analogy to the cell shape factor – in a square-shaped neighborhood of a STP $\mathbf{x}$ is computed according to the following formula:

$$I_r(\mathbf{x}) \quad = \quad \frac{1}{(2r+1)^2} \sum_{||\mathbf{y}-\mathbf{x}||_\infty \le r} \tfrac{1}{8}\, S(\mathbf{y}) \qquad (r = 0, 1, ...)$$

$S(\mathbf{y})$ measures the 'point surface' of $\mathbf{y}$, i.e. $S(\mathbf{y})$ is the number of direct neighbors of $\mathbf{y}$ belonging to another cell. We use 8-connectivity, hence $S(\mathbf{y})$ varies between 0 and 8. Using the maximum norm $||\cdot||_\infty$, the parameter r determines the size of a square around the point $\mathbf{x}$. For our networks we use r-values between 5 and 10. Note that $0 \le I_r(\mathbf{x}) \le 1$ for all STPs $\mathbf{x}$.
The above formula is only one of many possibilities to implement an irregularity measure. Future research will show whether there are more suitable variants.

4.4 Termination Rules

A general problem of heuristic search processes is the question of when to stop them. We have not investigated a problem-specific approach, but want to emphasize that this is an interesting open question. Early detection of low chance for good improvements can drastically reduce computation time by focussing computational resources on promising approaches.
The most common domain-independent termination rules are the *Max Rule* and the *Stagnation Rule.* The Max Rule defines a priori an upper bound on the number of search steps, whereas the Stagnation Rule observes the development of the solution cost and stops the process if over a predefined number of steps – the *lag interval* – the decrease of cost (measured in percent) drops below a given *critical threshold.*
Within the ARNO project, we used mainly the stagnation rule, because it realizes a good compromise between ease of implementation and adaptability to an individual process evolution.
It is noteworthy that the two parameters of the stagnation rule – the length of the lag interval and the critical threshold – are important *control* parameters. In general, stricter parameter values (shorter lag interval, higher threshold) lead to reduced average computation time, but also to lower average solution quality. So, depending on available resources and on performance requirements (both, average running time and solution quality), it is a *Meta-Optimization* problem to choose the parameters of the stagnation rule. Based on several experiments, parameter values in the intervals listed below have proven to be useful:

Parameter	Value Interval
lag interval	50 – 100 steps
threshold percentage	0 – 5 %

Table 2: Parameter Values for Stagnation Rule

5 Results

Within the ARNO project we have investigated eight real world problems. The data were provided by CNET [1]. The first four networks – denoted by N1_0, ..., N4_0 – are *greenfield design* problems, i.e. all candidate sites are initially unused. Networks N1_1, ..., N4_1 are *expansion design* problems, i.e. an existing solution should be expanded in order to meet new requirements, e.g. increased traffic demand.
We now present results for N1_0 and N3_0 in detail. N1_0 defines a highway scenario, N3_0 a medium size town scenario. The discussion of the expansion design problem is beyond the scope of this paper.

Network	Size	Candidate Sites	Total Traffic (Erlang)
N1_0	40 km × 170 km	250	3210.94
N3_0	50 km × 46 km	568	2988.08

Table 3: Network Data

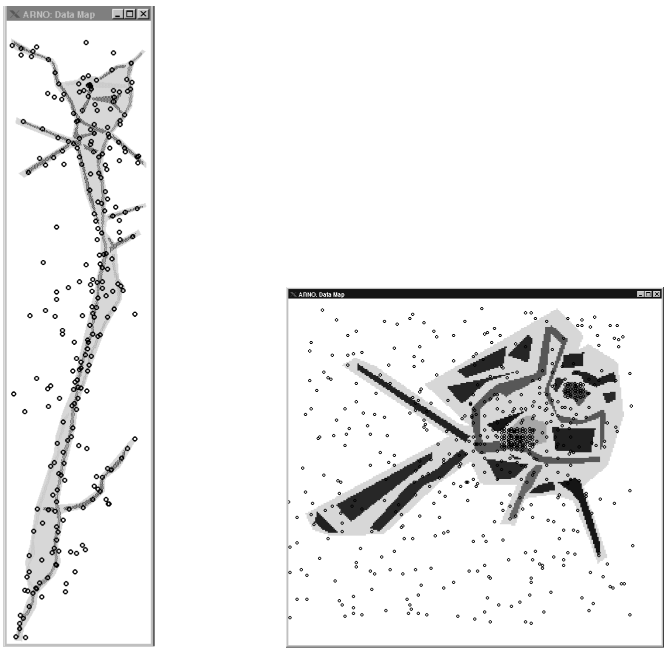

Figure 3: Data Maps of Network N1_0 and Network N3_0

Tables 4 and 5 display feasible solutions generated by our evolutionary search algorithm for Network N1_0 and Network N3_0. They have been obtained with the weights ψ_1, ψ_2, and ψ_3 given in the tables. The weights have been set so that one objective got 10 times more weight than the other two. The weights ϕ_1 and ϕ_2 of the hard cost function used in the repair phase have been set to 1.

[1]France Telecom Research and Development Center

The number of generations to compute these solutions lies between 1000 and 5000.

Best in...	# Sites (25)	IL	SF	OD	LD	SD	ψ_1	ψ_2	ψ_3
# Sites	38	70.0	6.1	2	6	102	10	1	1
IL	44	25.6	6.4	9	9	90	1	10	1
SF	43	113.3	5.3	4	31	84	1	1	10

Table 4: Best computed solutions for Network1_0

Best in...	# Sites (23)	IL	SF	OD	LD	SD	ψ_1	ψ_2	ψ_3
# Sites	30	100.5	9.3	1	0	87	10	1	1
IL	36	34.9	9.0	11	3	72	1	10	1
SF	32	162.0	6.8	0	7	84	1	1	10

Table 5: Best computed solutions for Network3_0

1. The *#Sites*-column contains the number of used sites. The number in parentheses is a lower bound for the number of needed sites resulting from the total traffic in a network.
2. The *IL*- and the *SF*-column display the interference level (in dBm per STP) and the average shape factor. Optimal shape factor in the euclidean geometry is 3.54, only reached by a circle. The shape factor of a hexagon is 3.72.
3. The *#OD*-, *LD*- and *#SD*-columns contain the number of used omnidirectional, large directive and small directive antennas, respectively.

Different weights lead to different solutions. There is a trade-off between the different objectives. We are currently trying to find a setting of the weights ψ_1, ψ_2, and ψ_3 leading to solutions which are a good compromise between the three objectives. This is currently done in cooperation with a network operator.

6 Conclusion

We have introduced an advanced model for the antenna placement problem, which addresses economical and several technical aspects. This leads to a constrained and multi-objective optimization problem, which we have tackled with an evolutionary algorithm. The results obtained so far are encouraging. Getting still better results is not a problem of the evolutionary algorithm, but of developing the used model. We are currently discussing with network operators how to extend the model in order to make it still more realistic.

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